The Development of Random Striations in the Positive Column of a Glow Discharge

I. GRABEC and S. POBERAJ
University of Ljubljana and The Jožef Stefan Institute

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The paper represents an analysis of the characteristic properties of spontaneously excited irregular striations observed in the positive column of a glow discharge. Comparisions with the properties of coherent, externally excited, growing ionization waves lead to the conclusion that irregular striations are random ionization waves. Assuming that random striations originate at the cathode end of the positive column, their frequency spectrum can be approximately explained from a knowledge of the dispersion relation of ionization waves. The nonlinear properties of random striations remain to be explained.

Introduction

Luminous standing or moving striations are perhaps the most cammon phenomenon observed in weakly ionized plasmas. Extensive investigations of artificially excited striations in the positive column of a glow discharge have partly cleared up the picture of the wave mechanism of moving striations [1, 2]. It is now believed that ionization waves, as striations are usually called, appear due to changes in the ionization rate caused by variations of electron temperature [2] or more strictly, by variations in electron velocity distribution [3]. Processes leading to the ionization of an atom in a weakly ionized plasma can be extremely complex. The theoretical description of ionization waves is therefore rather cumbersome and involves so many simplifications that anything more than a qualitative explanation of the instability, leading to the stratification of the positive column, could hardly be obtained. But there are still unexplained phenomena, connected with striations, which could be phenomenologically described and even understood without detailed knowledge about the instability. Such are for example, the development of random striations [4], the formation of a discontinuity in a wave of stratification [5], and the development of a turbulence in the positive column [6].

In the present article we attempt to explain the development of random striations in a positive column. This explanation is based on some data obtained experimentally and on rather general principles of random noise theory. In a previous paper [4] some comments on characteristic properties of fluctuations in the positive column of a glow discharge in neon were given. The same is done now for the positive column of an argon glow discharge in a longitudinal magnetic field, where random ionization waves can be also observed [7]. From the experimental data some conclusions about the source of the fluctuations and the sort of instability are then drawn which help to explain the shape of the frequency spectra of growing fluctuations. At the end some suggestions about future work are given.

Reprint requests to Dr. I. GRABEC, Fakulteta za Strojništvo, Univerza v Ljubljani, Askerčeva 16, *Ljubljana*, Jugoslavia.

The Properties of Random Striations

Selfexcited random striations in the positive column of a glow discharge frequently possess random character. They can be observed in broad regions of discharge parameters, especially in noble gases [1, 8, 9]. Measurements of dispersion properties of externally excited ionization waves show that a noisy positive column can support unstable waves. Therefore one would conclude that random striations are nothing else than ionization waves excited by some random source. Our aim is to find this connection.

The experiments were done on a 130 cm long pyrex discharge tube with inner diameter 2.2 cm. The electrodes were two cylinders made out of nickel. The distance between them was 120 cm. Ten centimeters in front of the cathode a ring electrode, made out of a tungsten wire, was mounted in the tube. It served for external excitation of ionization waves. In the middle of the tube two wire probes were inserted for measurement of the electric field intensity. The tube was filled with argon and placed along the axis of a 120 cm long solenoid. The magnetic field was applied because some characteristic parameters of ionization waves, like optimal increment, can be changed easily by varying the magnetic field [7]. At the same time the nature of ionization waves is not altered essentially if the magnetic induction is far below the critical value for the onset of helical instability [10, 11]. In our case this value is approximately equal $B_c = 2.6$ kG. The striations were detected by a moveable photo FET. The discharge was mentainend by a current stabilised rectifier. The experimental data presented here were obtained with the following discharge parameter values: discharge current $I = 10$ mA, argon gas pressure $p = 0.1$ Torr, magnetic induction $B = 1$ kG, electric field intensity $E = 2.8$ V/cm.

Without external excitation, random striations were present in the positive column and the oscilloscopic trace of a signal from the photodetector showed random noise. The root mean square amplitude N_{eff} of this noise grows when the detector is moving from the cathode to the anode. Its record, as a function of detector position along the tube, is shown in Figure 1. Three characteristic regions

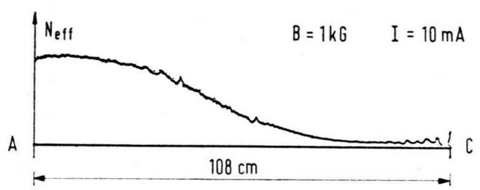

Fig. 1. The root mean square amplitude N_{eff} of a signal from the photodetector as a function of position. Vertically arbitrary units.

can be distinguished in the discharge in accordance with this picture: the low noise region, at the cathode, and the growth region as well as the saturated region in the positive column. More information about random striations follows from spectral analysis of signals from the photodetector. The frequency spectra obtained with different positions of the detector are shown in Fig. 2, where, as conveniently, $-z$ denates the distance from the cathode. It should be noted that the peak at the beginning of the spectrum is just a zero marker. Figure 2 shows that a relatively broad spectrum of fluctuations is obtained at the beginning of the growth region. From this spectrum a new one, of approximately Gauss-ian shape, develops in the growth region. Its maximum appears in the vicinity of 5 kHz, which should be called the optimal frequency. When going still further away from the cathode, first a new peak appears at 10 kHz and then a broader spectrum of rather stable shape is

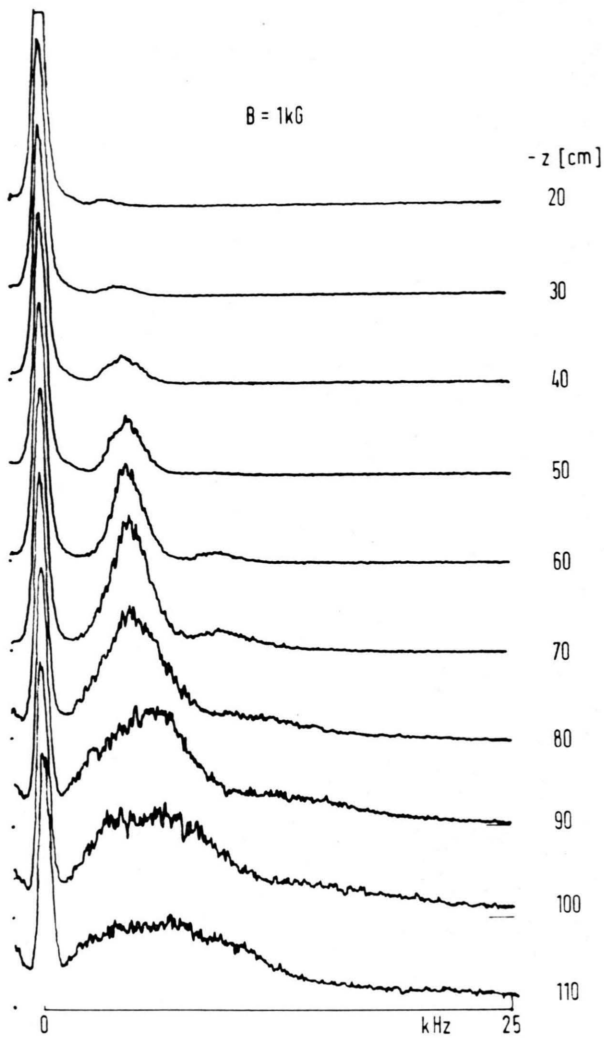

Fig. 2. The frequency spectra of the signal from the photodetector at different positions along the discharge. Vertically arbitrary units.

obtained in the saturated region. All these properties of fluctuations were also observed in a neon glow discharge [4]; only the lengths of characteristic regions and the optimal frequency were different.

Dispersion Properties of Externally Excited Waves

To find the connection between random striations and ionization waves we proceed to determine the dispersion relation of the latter. For this purpose the ionization waves were excited by voltage signals, of various constant frequencies, on the ring electrode. During the motion of the photodetector along the tube, the signal from it was analysed with a phase sensitive amplifier. A record of the output at the optimal frequency is shown in Fig. 3 as a function of position. With it the record of the effective amplitude of random striations is represented. The first record shows the growing wave in an appreciable part of the discharge. The wave amplitude grows when going away from the source, which shows that the positive column is convectively unstable. In our case the amplitude of the launched wave is approximately equal to the effective amplitude of random fluctuations nearly everywhere except in the saturated region. In this region the synchronous wave begins to be damped. If the amplitude of the launcher is slightly raised, saturation appears nerare to the cathode, but the wave length and the spatial increment in the lower part of the growth region of the wave are not altered. They are also not changed if an another wave of approximately equal amplitude and different frequency is simultaneously excited. But in this case nonlinear mixing of both waves is observed in the upper part of the growth region [6]. From all this we can conclude that in the lower part of the growth region the positive column exhibits the properties of a linear amplifier. This is, however, true only for the launched wave with an amplitude far below the effective amplitude of random fluctuations in the saturated region. Of course all waves are not growing. If the frequency is shifted from the optimal value the increment is lowered and even changes its sign. The dependence of the wave number k_r and the spatial increment k_i on the frequency, as measured in the lower part of the growth region, is shown in Figures 4 and 5. The measurements were done only on the anode side of the launcher. On the cathode side no growing waves were observed. It should be pointed out that the phase velocity of externally excited waves is oriented to the cathode, that is in the positive direction. The monochromatic ionization wave in the lower part of the growth region on the anode side of the launcher can be thus described by the real part of the expression

$$n = n_0 \exp\{-i\,\omega\,t + i\,k_r(\omega)\,z - k_i(\omega)\,z\}\,. \quad (1)$$

Here $z=0$ corresponds to the position of the launcher and $z<0$ to its anode side, n is the wave amplitude, ω is the circular frequency and $k(\omega) = k_r(\omega) + i\,k_i(\omega)$ is a complex wave number. From Fig. 4 the group velocity of ionization waves is seen to be negative and the ionization waves under examination are therefore of backward character. All mentioned dispersion properties of ionization waves are in agreement with some other observations [2]. One of the main characteristics of ionization waves in our case is that they are convectively unstable, which means that the amplitude of the disturbance grows when it moves toward the anode with group velocity.

Let us now discuss some similarities between random striations and coherent ionization waves. Perhaps the most striking observation is that the maximum of the noise spectrum appears at the same frequency as the maximum of the spatial increment of coherent waves. The effective amplitude of both sorts of oscillations grows when going toward the anode. Using correlation techniques, as proposed by several authors [8, 9], the backward character of randon striations can be found. This also agrees with the property of ionization waves. We therefore conclude that random striations are stochastic ionization waves. The question of how they are excited is immediately at hand. The origin of random striations has not been investigated. It seems that random fluctuations are transmitted from the regions near the cathode to the positive column where they are amplified due to the instability. In principle any part of the discharge can act as a noise source. But even in the case where the noise sources are unifomly distributed along the discharge, the highest fluctuations would on average originate at the cathode side of the positive column. An oversimplified model, describing the noisy discharge presented in this paper, would then consist of an unstable positive column and a noise source, with broad frequency spectrum, localized at the cathode end of the column. In the most simple case the spectrum would be taken to be white. We are now going to find the frequency spectrum of fluctuations described by such a model. As only linear properties of ionization waves have been examined, the spectrum of the fluctuations in the lower part of the growth region will be calculated.

The Frequency Spectrum of Convectively Unstable Ionization Waves Excited by a Localized Random Source

In this part of the article we try to derive those characteristic properties of growing random striations which follows directly from a knowledge of the dispersion properties of ionization waves and the properties of the source. We assume that the positive column may be treated as a linear amplifier. The characteristic observable $n(z,t)$ can be then expressed by a source function $q(z,t)$ and a weighting, or Green's function, $G(z,t)$ by the expression:

$$n(z,t) = \int G(z-z', t-t')\, q(z',t')\, dz'\, dt'. \quad (2)$$

When the source is random, the response function is also random. To find its frequency spectrum we first put down an equation for the correlation function:

$$\begin{aligned} R_n(z_1,t_1,z_2,t_2) &= \langle n(z_1,t_1)\, n(z_2,t_2) \rangle \quad (3) \\ &= \int G(z_1-z_1', t_1-t_1')\, G(z_2-z_2', t_2-t_2') \\ &\quad \cdot \langle q(z_1',t_1')\, q(z_2',t_2') \rangle \cdot dz_1'\, dz_2'\, dt_1'\, dt_2'. \end{aligned}$$

This equation is obtained by multiplying two equations of the form (1) at two different space-time points and then averaging over some representative ensemble. The response function is not a random quantity and can be therefore taken out of the brackets denoting the average. We further assume that the noise source is stationary. In this case the correlation function can be written in the form

$$\langle q(z_1,t_1)\, q(z_2,t_2) \rangle = R_q(t_1-t_2)\, \delta(z_1)\, \delta(z_2) \quad (4)$$

and Eq. (3) is transformed to

$$\begin{aligned} R_n(z_1,t_1,z_2,t_2) &= \int G(z_1, t_1-t_1')\, G(z_2, t_2-t_2') \\ &\quad \cdot R_q(t_1'-t_2')\, dt_1'\, dt_2'. \quad (5) \end{aligned}$$

Instead of the source correlation function we prefer to give its frequency spectrum, defined by the inverse of the Fourier transform

$$R_q(t_1-t_2) = S_q(\omega) \exp\{-i\,\omega(t_1-t_2)\}\, d\omega\,. \quad (6)$$

When the system is convectively unstable the Fourier transform with respect to time exists also for the Green's function [12]

$$G(z,t) = \int g(z,\omega) \exp\{-i\,\omega\,t\}\, d\omega\,. \quad (7)$$

From the reality of the Green's function it follows

$$g^*(z,\omega) = g(z,-\omega). \quad (8)$$

Taking into account Eqs. (6) and (7) and integrating with respect to times and frequencies we get from Eq. (5)

$$\begin{aligned} R_n(z_1,t_1,z_2,t_2) &= (2\pi)^2 \\ &\quad \cdot \int g(z_1,\omega)\, g^*(z_2,\omega)\, S_q(\omega) \quad (9) \\ &\quad \cdot \exp\{-i\,\omega(t_1-t_2)\}\, d\omega\,. \end{aligned}$$

For the correlation function at some position $z = z_1 = z_2$ we obtain

$$R_n(z,t_1,z,t_2) = \int S_n(z,\omega) \exp\{-i\,\omega(t_1-t_2)\}\, d\omega \quad (10)$$

where the spectrum of the signal n is given by

$$S_n(z,\omega) = (2\pi)^2\, |g(z,\omega)|^2\, S_q(\omega). \quad (11)$$

This expression is similar to the well known Wiener-Lee input-output relation for a discrete amplifier [13].

We are now in a position to find the frequency response function $g(z,\omega)$ by taking into account experimentally obtained results concerning the response of a positive column to external perturbations. It follows first from the inverse Fourier transform of Equation (7):

$$g(z,\omega) = (2\pi)^{-1} \int G(z,t) \exp\{i\,\omega\,t\}\, dt\,. \quad (12)$$

In the previous part of the article we found that the wave described by Eq. (1) appears on the anode side of the localised source of given constant frequency. The source function in this case is

$$q(z,t) = q_0\, \delta(z) \exp\{-i\,\omega\,t\} \quad (13)$$

where q_0 is the source amplitude. With this in mind we get from Eq. (2), using the experimental result Eq. (1),

$$\begin{aligned} n_0 \exp\{i\,k(\omega)\,z - i\,\omega\,t\} \\ = q_0 \int G(z,t-t') \exp\{-i\,\omega\,t'\}\, dt' \end{aligned} \quad (14)$$

and after some rearrangement,

$$\begin{aligned} n_0\, q_0^{-1} \exp\{i\,k(\omega)\,z\} \\ = \int G(z,t) \exp\{i\,\omega\,t\}\, dt = 2\pi\, g(z,\omega). \end{aligned} \quad (15)$$

The frequency response function is thus proportional to $\exp\{i\,k(\omega)\,z\}$ and we can put down

$$g(z,\omega) = g(\omega) \exp\{i\,k(\omega)\,z\}\,. \quad (16)$$

This equation adequately describes our observations and holds only on that side of the source where growing waves are obtained. Inserting Eq. (16) into Eq. (11) we get for the frequency spectrum of random ionization waves, at some place with $z<0$, the following expression:

$$S_n(z,\omega) = S_0(\omega) \exp\{-2\,k_i(\omega)\,z\}\,. \quad (17)$$

Here $S_0(\omega)$ is a proportionality factor corresponding to a frequency spectrum of the fluctuations at the place of excitement. Let us now discuss some characteristic properties of the spectrum (17). The spatial increment $k_i(\omega)$ has a maximum at a certain optimal circular frequency ω_0. Using the Taylor expansion we can write

$$k_i(\omega) = k_{i0} + k''_{io}(\omega - \omega_0)^2/2 + \dots \tag{18}$$

where index 0 denotes optimal value, and two primes the second derivative with respect to ω. Inserting this into Eq. (17) we get an approximation:

$$S_n(z, \omega) = S_0(\omega) \cdot \exp\{-2\, k_{i0}\, z - k''_{io} z (\omega - \omega_0)^2\} \tag{19}$$

holding for $z < 0$. Remember also that k''_{io} is negative!

If the spectrum $S_0(\omega)$ is a mild function of frequency, and it should be in the case of a white noise source, the spectrum S_n has a Gaussian shape at some distance on the anode side of the noise source. Its maximum appears at the optimum frequency and grows when going toward the anode. The average power of the fluctuations is obtained by integrating $S_n(z, \omega)$ with respect to ω. If the function $S_0(\omega)$ is again considered to be slowly varying in the vicinity of ω_0, the following space dependence is obtained:

$$\int S_n(z, \omega)\, d\omega \sim |z|^{-1/2} \exp\{-2\, k_{i0}\, z\}. \tag{20}$$

The effective amplitude of fluctuations is given by the square root of this value and is proportional to

$$N_{\text{eff}} \sim |z|^{-1/4} \exp\{-i\, k_{i0}\, z\}. \tag{21}$$

This function is similar to $\exp\{-i\, k_{i0}\, z\}$, for large $|z|$ and the effective amplitude of fluctuations grows like that of the most unstable wave. This result was also found experimentally and is represented in Figure 3.

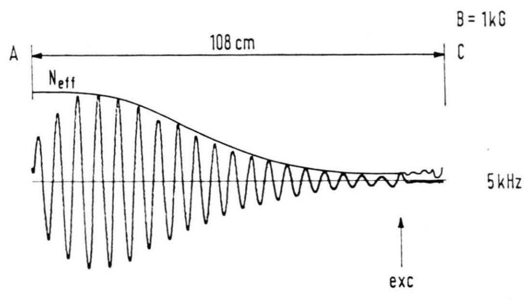

Fig. 3. The record of a coherent wave amplitude as measured by the phase sensitive detector. Upper curve N_{eff}.

Let us now mention that similar dispersion cources like those represented in Figs. 4 and 5 can also be measured at different discharge conditions or even predicted theoretically [1, 2, 14].

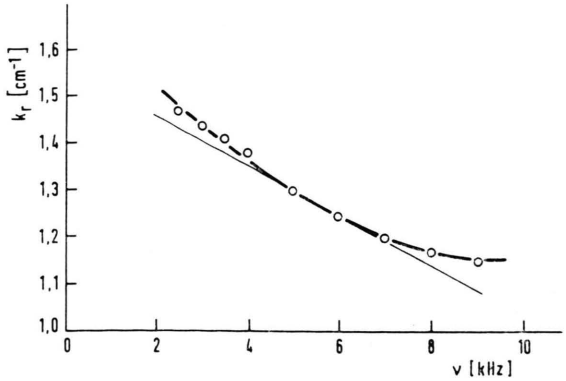

Fig. 4. The wave number of ionization waves versus frequency. The group velocity $v_g = -1.2 \cdot 10^3$ ms^{-1} is obtained at the characteristic wave number $k_{r0} = 1.3$ cm^{-1}. The characteristic phase velocity is $v_{p0} = 2.4 \cdot 10^2$ ms^{-1}.

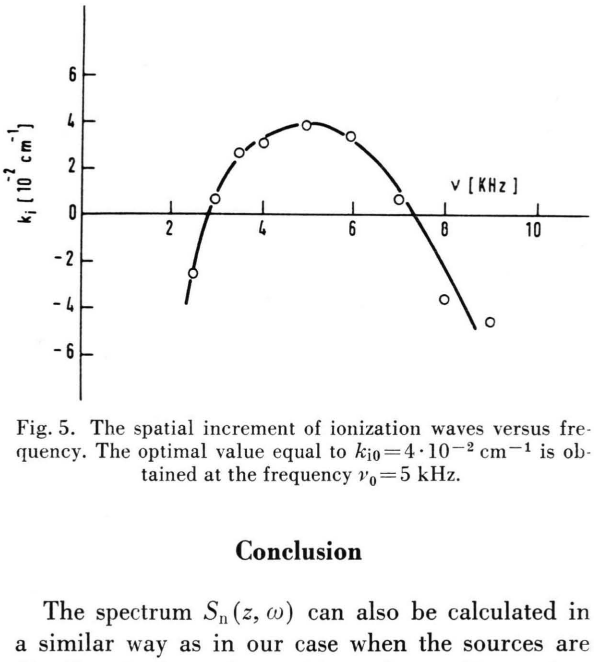

Fig. 5. The spatial increment of ionization waves versus frequency. The optimal value equal to $k_{i0} = 4 \cdot 10^{-2}$ cm^{-1} is obtained at the frequency $\nu_0 = 5$ kHz.

Conclusion

The spectrum $S_n(z, \omega)$ can also be calculated in a similar way as in our case when the sources are distributed along the positive column. For such a calculation the source correlation function R_q, appearing in Eq. (3), must be known. Up to now there are no experimental data available about its properties. Also the influence of strong fluctuations in the cathode region has not been examined. Even in the case where the correlation function of these fluctuations is known, their contribution to the noise in the positive column could hardly be taken into account because the dispersion properties of the transition regions between the cathode and the positive column are not known.

The measured spectra of the fluctuations in their linear region of a positive column can be satisfactorily described by the above calculations. The only necessary assumption is that the fluctuations originate from some localised source with a broad frequency spectrum. The main task of future work would be an adequate description of the transition and saturated region. It is obvious that in these regions nonlinear effects play a significant role. Some experimental investigations of nonlinear striations have already given interesting results [6, 9].

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Epitaxie von dünnen Tellurschichten auf Alkalihalogeniden bei 100 °C

Egon Dick und Werner Schüz, Mosbach (Baden) *

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Epitaxy of Thin Layers of Tellurium on Alkalihalogenides at 100 °C

Te was evaporated in high vacuum (10^{-5} Torr) on simultaneously cleavaged alkali halides with rock salt structure. Research on the Te-films was done by transmission electron microscopy. We observed characteristic differences in epitaxy quality of the different substances: e. g. NaCl and LiBr showed bad, RbJ and KJ showed exceptionally good epitaxy (exclusively point diffraction patterns). Reproduced experiments showed the same results as simultaneous evaporation. The expected systematic relation to analoguous *d*-values was however insignificant. It seems to be an "indirect epitaxy" caused by intermediate layers of different structures.

1. Bedingungen für das Auftreten der Epitaxie

Über die Bedingungen für die Epitaxie besteht bis heute noch keine einheitliche Vorstellung. Die Forderung nach einer zweidimensionalen Strukturanalogie [1, 2] ist keine notwendige Bedingung: Auch Abweichungen der beiden Gitter um über 14% (Grenze nach [3, 4]) verhindern Epitaxie nicht. Bei Erwärmung der Unterlage auf „Epitaxietemperatur“ [5] ist sogar Epitaxie von Ag-(100) auf NaCl-(100) zu beobachten, obwohl hier die Gitterabweichungen schon 26% betragen. Je nach Versuchsbedingungen schwankt die Epitaxietemperatur jedoch erheblich. Die naheliegende Vermutung, daß Gasbeladung oder Verschmutzung der Oberfläche des Wirtskristalles verantwortlich sei, führte zu Aufdampfversuchen im Ultrahochvakuum [6, 7]; Die Epitaxie verschlechterte sich jedoch. Die Gasart im Spalt- und Aufdampfmilieu spielt offensichtlich eine wichtige Rolle [7, 8]: Wasserdampf fördert in bestimmten Mengen (kritische Bedeckung der Substratflächen) die Epitaxie. Möglicherweise werden durch eine geeignete Adsorptionsschicht Potentialmulden ausgebügelt, wodurch die Oberflächenbeweglichkeit der Ag-Atome erhöht wird. Es ist aber ebensogut möglich, daß die Wasserdampfmoleküle in die Substraoberfläche eingebaut werden und eine hydratartige Zwischenschicht bilden, die in ihren Gitterabmessungen besesr zu den Gitterabmessungen der Aufdampfsubstanz paßt [9].

* Aus dem Physikalischen Laboratorium Mosbach, in Verbindung mit der Universität (TH) Karlsruhe (V 148/71).

2. Aufgabe und Versuchsmethode

Trotz einer Vielzahl von Untersuchungen und Deutungsbemühungen ist nur sichergestellt, daß das Epitaxieproblem sehr komplex ist. Es schien uns da-